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GASDYNAMIC PARTICLE ACCELERATORS - AN OPTIMIZATION,(U)
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(6) GASDYNAMIC PARTICLE ACCELERATORS - AN OPTIMIZATION

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Prepared by: E. Migotsky
(10) E. Migotsky
Senior Staff Engineer
Aero/Thermodynamics and Nuclear Effects

Approved by: [signature]
H. Hurwicz
Chief Advance Technology Engineer
Aero/Thermodynamics and Nuclear Effects
Research and Development
Advance Systems and Technology

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MCDONNELL DOUGLAS ASTRONAUTICS COMPANY-WEST

5301 Bolsa Avenue, Huntington Beach, CA 92647

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SUMMARY

The problem of determining the streamwise variation of gas stream velocity in a nozzle such that the acceleration of particles within the gas stream is everywhere a maximum has been reduced to a quadrature when the following parameters are constants: ratio of specific heats and stagnation conditions of the gas; mass, size, and drag coefficient of the particles. The quadratures have been evaluated for three values of the ratio of specific heats: 5/4, 7/5, and 5/3.

The results indicate that a high-enthalpy, high-pressure plasma arc with an optimized nozzle appears to offer the possibility of significantly improved simulation of high heating rates simultaneously with high particle velocities and longer testing times required for advanced hypersonic vehicles. For example, assuming a particle drag coefficient of 1.0, a plasma arc with a stagnation enthalpy of 6,000 Btu/lbm and a stagnation pressure of 190 atmospheres may achieve particle velocities of 11,800 and 7,000 feet per second for particle diameters of 50 and 1,500 microns, respectively, with a 17.25 foot long optimized nozzle.

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SYMBOLS

A	nozzle area
a	speed of sound
C_D	particle drag coefficient
d	diameter
h	enthalpy
M	Mach number
m	mass
p	pressure
S	reference area
s	distance
t	time
V	velocity
v	reduced velocity, V/V_m
x	non-dimensional distance, s/λ
γ	ratio of specific heats
λ	characteristic length defined by Equation (7)
ρ	density

Subscripts and Superscripts

f	nozzle exit
m	gas stream maximum
p	particle
s	gas stream
t	gas stagnation conditions
$*$	sonic flow conditions

Section 1

INTRODUCTION

The rational design of a hypersonic vehicle requires the ability to predict the behavior of the vehicle operating within the environment to which it is exposed. This environment may include high speed flight through an atmosphere containing rain, ice and/or dust particles which results in both particle erosion as well as ablation of the heat-protection system. Neglecting the erosion due to particles can result in destruction of the vehicle with resultant failure of the mission. Computer programs of various degrees of sophistication have been developed at MDAC, and by other investigators, to predict the behavior of thermostructural systems in such environments. These programs require empirical erosion coefficients which have been determined from simulated ablation-erosion experiments.

Existing hypervelocity erosion facilities do not simulate the real flight environment completely and particularly for the advanced vehicles. The short-comings may consist of one or more of the following characteristics: too low velocities, too short exposure times, lack of simultaneous heating and erosion simulation, inadequate means of dispersing the particles, inadequate size of model or particles or both. Some of the methods used to obtain erosion data include the following: (1) accelerating a small group of particles by means of a shock tunnel ("hot-shot") or a light-gas gun and impinging these particles upon a specimen which has been preheated by a plasma arc or by a radiant heater; (2) firing the specimen down a ballistics range (maximum test length less than 1,000 ft) and recording data in flight; (3) rocket sled testing which is expensive, is limited to velocities below flight velocities, and has not been developed to the point of obtaining erosion data with small dust particles; (4) hypersonic

wind tunnels into which dust particles are injected provide more realistic testing times and multiple impingement but low velocities, low heating, and low shear. All these methods provide widely varied, and limited levels of erosion simulation. A definite need exists for more realistic simulation of the flight environment which can result in combined ablation and particle erosion.

Of the current methods for conducting erosion tests only the wind tunnels can provide testing times of sufficient duration to match flight times (of the order of 10 seconds). The characteristics of wind tunnels which result in low velocities and low heating are the relatively low values of stagnation enthalpy and stagnation pressure associated with such facilities. These limitations may be overcome by utilizing another facility as the energy source for a particle accelerator, namely, the plasma arc which has been used extensively for ablation testing. In particular, if we inject the dust particles at or near the throat of a high-enthalpy, high-pressure plasma arc, then utilize a high Mach number nozzle to accelerate the gas stream to high velocities, the gas stream will accelerate the dust particles to velocities that can approach the stream velocity. A facility of this type would provide a reasonable testing time together with higher particle velocities and larger heating rates than current wind tunnels.

Before embarking on the design and hardware effort of modifications to existing arc facilities, it was felt that an analytical investigation was mandatory to explore the phenomena involved in the energy transfer, from the internal energy of the gas, to the kinetic energy of the particles. A primary objective was to investigate methods of tailoring the gas flow in order to obtain efficient acceleration of the particles, in particular, to determine methods for optimizing

such gasdynamic particle accelerators. The results of this investigation should provide the methods for an assessment of the capabilities of existing wind tunnels and arc jets as particle accelerators and would provide the means for a rational approach to the modification of existing facilities as well as the design of new facilities for conducting steady-state dust erosion experiments and combined ablation-erosion experiments. Moreover, the availability of an optimum solution, even if somewhat idealized, is useful for many reasons. Some of these include: a goal to which a designer can strive; a basis for evaluating an existing design; a focus upon the limitations that nature imposes upon a problem; and, more often than not, an indication of the modifications that would improve the performance of an existing design.

Section 2

ANALYSIS

2.1 Equations of Motion

The basic equation of motion for a particle in a gas stream, such as a wind tunnel, arc-jet, or nozzle, may be written in the form

$$\frac{C_D \rho_s S_p (V_s - V_p)^2}{2} = m_p \frac{dV_p}{dt} \tag{1}$$

Equation (1) may also be written, by noting that $dV_p/dt = (dV_p/ds)(ds/dt) = V_p\, dV_p/ds$ where s is the arc length along the trajectory of the particle,

$$\frac{C_D \rho_s S_p (V_s - V_p)^2}{2} = m_p V_p \frac{dV_p}{ds} \tag{2}$$

The general one-dimensional problem of particle motion in a gas stream corresponds to specifying the gas density, $\rho_s(s)$, and the stream velocity, $V_s(s)$, as arbitrary functions of distance along the gas stream and then integrating equation (2) to obtain the particle velocity, V_p, as a function of the distance, s, or of the time, t, (in equation [1]).

For a perfect gas, with isentropic flow, the gas density may be related to the stream velocity, V_s, and the stagnation density, ρ_t, through the relation (reference 1)

$$\frac{\rho_s}{\rho_t} = \left[1 - \frac{\gamma-1}{2}\left(\frac{V_s}{a_t}\right)^2\right]^{\frac{1}{\gamma-1}} \tag{3}$$

Substituting equation (3) into equation (2) and dividing by a_t^2 gives

$$\left[\frac{C_D S_p \rho_t}{2\ m_p}\right]\left[1 - \frac{\gamma-1}{2}\left(\frac{V_s}{a_t}\right)^2\right]^{\frac{1}{\gamma-1}}\left(\frac{V_s}{a_t} - \frac{V_p}{a_t}\right)^2 = \left(\frac{V_p}{a_t}\right)\frac{d\ (V_p/a_t)}{ds} \qquad (4)$$

Note that for any gas stream (with specified stagnation conditions) there exists a maximum velocity, V_m, that is assocaited with the given stream. The velocity corresponds to the velocity attainable by exapnding the gas to zero pressure and is related to the stagnation speed of sound, a_t, by the relation

$$a_t^2 = V_m^2\left(\frac{\gamma-1}{2}\right) \qquad (5)$$

Substituting equation (5) into equation (4) gives

$$\left[\frac{C_D S_p \rho_t}{2\ m_p}\right]\left[1 - \left(\frac{V_s}{V_m}\right)^2\right]^{\frac{1}{\gamma-1}}\left(\frac{V_s}{V_m} - \frac{V_p}{V_m}\right)^2 = \left(\frac{V_p}{V_m}\right)\frac{d(V_p/V_m)}{ds} \qquad (6)$$

Substituting the nondimensionalized velocities v_s and v_p, and noting that the quantity

$$\lambda = \left[\frac{2\ m_p}{C_D S_p \rho_t}\right] \qquad (7)$$

has the dimension of length, we can nondimensionalize the distance s by dividing by λ, so that $x = s/\lambda$ and equation (6) becomes (when λ is a constant)

$$\left(1 - v_s^2(x)\right)^{\frac{1}{\gamma-1}}\left(v_s(x) - v_p\right)^2 = v_p\,\frac{dv_p}{dx} \qquad (8)$$

The problem of evaluating the effectiveness of a gasdynamic stream for accelerating particles is imbedded within the solutions of equation (8). The

range of variables of interest consists of nondimensional velocities bounded by $0 \leq v_p \leq v_s \leq 1$. The direct problem consists of obtaining solutions to equation (8) for specified variations of the nondimensional gas velocity $v_s(x)$ as a function of the distance. It should be noted that equation (8) can be modified to include the case of gasdynamic decelerators ($v_p > v_s$) by multiplying the right hand side by minus one. It should also be noted that rather large ranges of physical situations are included in the apparently simple format of equation (8). These physical conditions include the following: arbitrary particle size (e.g., from less than 1 micron in diameter to well over 1 centimeter); the complete range of stagnation conditions, such as, pressure, density, and pressure (within the range of a perfect gas); and the full range of stream Mach number from zero to infinity.

A preliminary assessment of the character of equation (8) which governs the motion of particles in an accelerating gas stream indicates that its apparent simplicity is deceiving. The equation, although of first order, is nonlinear and with variable coefficients. The analytical solution of such equations is not, in general, an easy task. The particular equation with which we are confronted here can be shown to be of the Riccati type. Such equations, in the general case, are not reducible to quadratures. One approach to the analytical solution of the basic equation of particle motion in a gas stream would be to make judicious approximations in order to render the equation tractable while retaining the essence of the problem. This method was used in reference 2 and reasonable results appear to have been obtained. Another approach is to seek exact solutions to special problems. In this approach two categories of special problems are available. The first is to seek solutions to indirect problems in which the particle motions ($v_p(x)$) are specified and then to solve for the stream

velocity ($v_s(x)$) required to obtain the given motion. This approach circumvents the solution of the nonlinear differential equation; however, the success of this method depends upon the astuteness of the specified motion and its relation to physical reality. The more interesting approach is to seek optimization solutions. The present investigation, therefore, focussed upon the following optimization problem: For every particle velocity, v_p, find the value of stream velocity, v_s, that maximizes the acceleration of the particle. Such a solution gives the shortest distance to accelerate the particles from one value of velocity to another value.

2.2 Optimization

In order to determine the stream velocity, v_s, that maximizes the acceleration for every given particle velocity, the left hand side of equation (8) is differentiated with respect to v_s, and the derivative is set equal to zero. The resulting equation is

$$\frac{-2v_s}{\gamma-1}\left(1-v_s^2\right)^{\frac{2-\gamma}{\gamma-1}}(v_s-v_p)^2 + 2\left(1-v_s^2\right)^{\frac{1}{\gamma-1}}(v_s-v_p) = 0 \qquad (9)$$

One solution to equation (9) is $v_s = v_p$, the uninteresting case for which the particle acceleration vanishes. The other solution is readily obtained from equation (9) by solving for v_p, thus,

$$v_p = \gamma v_s - \frac{(\gamma-1)}{v_s} \qquad (10)$$

Equation (10) provides the desired relation between the dimensionless stream and particle velocities for which the particle acceleration is a maximum. The

associated maximum nondimensional particle accleration, $v_p \frac{dv_p}{dx}_{max}$, is obtained by substituting equation (10) into equation (8) to give

$$\left(v_p \frac{dv_p}{dx}\right)_{max} = \frac{(\gamma-1)^2 \left(1-v_s^2\right)^{\frac{2\gamma-1}{\gamma-1}}}{v_s^2} \tag{11}$$

Although equation (10) represents a significant step in the optimization problem by relating the two velocities, there reamins to relate these velocities to some geometric parameter such as the nondimensional distance, x, in the direction of the flow; i.e., the functions $v_s(x)$ and $v_p(x)$ are to be determined. One approach is to solve equation (10) for v_s, substitute the result into equation (8) to obtain a differential equation for v_p, and then solve the differential equation. This procedure results in integrals that are difficult to evaluate. A more fruitful approach was found to be to transform equation (10) into a differential equation in v_s in the following manner. Differentiate equation (10) with respect to x to obtain

$$\frac{dv_p}{dx} = \frac{dv_s}{dx}\left[\gamma + \frac{(\gamma-1)}{v_s^2}\right] \tag{12}$$

Then, substitute equations (10) and (12) into equation (8) to obtain, after some algebraic manipulation, the following differential equation

$$\frac{v_s \left(1-v_s^2\right)^{\frac{2\gamma-1}{\gamma-1}}}{\left(\frac{\gamma}{\gamma-1}\right)^2 v_s^4 - 1} = \frac{dv_s}{dx} \tag{13}$$

This differential equation can be solved for x by separation of variables to give

$$x = \int_{v_s(0)}^{v_s(x)} \frac{\left[\left(\frac{\gamma}{\gamma-1}\right)^2 v^4 - 1\right] dv}{v\left(1-v^2\right)^{\frac{2\gamma-1}{\gamma-1}}} \tag{14}$$

Thus, the problem of determining the variation of dimensionless stream velocity with dimensionless distance along the flow direction to obtain maximum particle acceleration has been reduced to a quadrature. The associated particle velocities may then be obtained from equation (10).

The integrals in equation (14) have been evaluated using the tables of references (3) and (4) for three constant values of the ratio of specific heats, γ; namely, 5/4, 7/5, and 5/3. These values of γ are representative, respectively, of air in the nozzle of a high temperature air arc, a perfect diatomic gas, and a perfect monatomic gas. After considerable algebraic manipulation, the expressions for x as a function of v_s were reducible to the following forms:

for $\gamma = 5/4$:

$$x = \frac{1}{\left(1-v_s^2\right)^5}\left(-\frac{53}{30} + \frac{19v_s^2}{3} - \frac{47v_s^4}{12} + \frac{9v_s^6}{4} - \frac{v_s^8}{2}\right) + \ln\left|\frac{\left(1-v_s^2\right)^{1/2}}{v_s}\right| + 1.258 \tag{15}$$

for $\gamma = 7/5$:

$$x = \frac{1}{\left(1-v_s^2\right)^{7/2}} \left(- \frac{499}{210} + \frac{379v_s^2}{60} - \frac{10v_s^4}{3} + v_s^6 \right) \tag{16}$$

$$+ \ln \left| \frac{1+\left(1-v_s^2\right)^{1/2}}{v_s} \right| + 1.425$$

for $\gamma = 5/3$:

$$x = \frac{1}{\left(1-v_s^2\right)^{5/2}} \left(- \frac{71}{30} + \frac{53v_s^2}{12} - v_s^4 \right) + \ln \left| \frac{1+\left(1-v_s^2\right)^{1/2}}{v_s} \right| + 1.694 \tag{17}$$

Section 3

RESULTS AND DISCUSSION

The relation between reduced particle velocity, v_p, and reduced stream velocity, v_s, for maximum particle acceleration is shown in Figure 1 for several values of the ratio of specific heats, γ. Although physically unrealistic, the case of $\gamma = 1$ is included to delineate a limiting case which corresponds to zero particle acceleration (since $v_s = v_p$). The value of v_s when $v_p = 0$ is of some interest since it corresponds to the stream velocity that gives maximum acceleration to the particle when the particle is injected into the stream with zero initial velocity. In this particular case the dynamic pressure acting upon the particle is identically equal to the dynamic pressure of the gas stream. This value of v_s is readily obtained from equation (10) by setting $v_p = 0$ to give

$$v_s = \left(\frac{\gamma-1}{\gamma}\right)^{1/2} \quad \text{when } v_p = 0 \tag{18}$$

Since the Mach number, M_s, of the gas stream is related to the reduced stream velocity, v_s, by

$$M_s^2 = \left(\frac{2}{\gamma-1}\right) v_s^2 \left[1-v_s^2\right]^{-1} \tag{19}$$

It is easily shown that

$$M_s = \sqrt{2} \quad \text{when } v_p = 0 \tag{20}$$

for maximum particle acceleration. Moreover, this result is valid for all values of the ratio of specific heats, γ. Thus, in order to maximize particle

acceleration, the appropriate location for injecting particles with zero initial velocity into a nozzle is the point at which the gas stream Mach number is equal to $\sqrt{2}$.

The variation of reduced maximum particle acceleration, $\left[v_p \frac{dv_p}{dx}\right]_{max}$, with reduced stream velocity is presented in Figure 2 for three values of γ. Also included in Figure 2 is a dashed curve for the variation of maximum particle acceleration when the particle velocity is zero. The expression for this curve can be derived by substituting equation (18) into equation (11) to obtain

$$\left[v_p \frac{dv_p}{dx}\right]_{max} = (\gamma-1)\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} \quad \text{when } v_p = 0 \qquad (21)$$

Equation (21) gives the largest values of reduced particle acceleration for the problem considered in the present investigation; namely, the acceleration of particles with zero or finite velocity in the direction of a flowing gas. It is possible to have larger magnitudes of particle acceleration by injecting the particles upstream into the flowing gas ($v_p<0$); the deceleration of particles is not, however, the concern of the present investigation. It will be noted from Figure 2 that the reduced acceleration increases with increasing γ. Also from Figure 2 it is clear that the maximum particle acceleration decreases rapidly as the reduced stream velocity approaches the limiting gas velocity ($v_s \rightarrow 1.0$). In view of these low values of maximum particle acceleration it would be expected that nozzles designed as particle accelerators will become much longer as one requires the particle velocity to approach more closely the limiting gas velocity.

Another illustration of the variation of reduced particle acceleration with reduced velocities (particle velocity and stream velocity) is shown in

Figure 3 where lines of constant $(v_p \frac{dv_p}{dx})$ are plotted on the (v_p, v_s) - plane for $\gamma = 7/5$. Also included in Figure 3 is the optimum variation of v_p and v_s (repeated from Figure 1). It will be noted that the acceleration is flat in the neighborhood of the optimum curve. In addition the acceleration vanishes when $v_s = 1$ and when $v_p = v_s$. Thus, a nozzle design that deviates significantly from the optimum curve, either to the right or left of the optimum, will be significantly less effective in accelerating particles. In the region above and to the left of the optimum curve the particle slip velocity $(v_s - v_p)$ is too low although the stream density retains a significent value. In the region below and to the right of the optimum curve the nozzle is overexpanded and the density is too low, although the slip velocity may remain large. In either case, the dynamic pressure acting on the particle is less than the maximum. Inasmuch as the curves for constant reduced acceleration are flat in the neighborhood of the optimum, minor deviations from the optimum may not result in significant reductions in particle acceleration. Plots of the form of Figure 3, i.e., the (v_p, v_s) - plane, may be used as a guide for modifying an existing nozzle design to improve its ability to accelerate particles. In particular, if one plots the performance of a given design as a curve on the (v_p, v_s) - plane and compares it with the optimum curve, the qualitative changes in nozzle design become apparent: expand more rapidly those sections of the nozzle that lie to the left of the optimum curve and reduce the expansion ratio for the parts of the nozzle represented by a curve to the right of the optimum.

The variation of the reduced velocities, v_s and v_p, with dimensionless distance, x, along the flow direction to obtain maximum particle acceleration is presented in Figure 4 for three values of the ratio of specific heats, γ, equal to 5/4, 7/5,

and 5/3. The semi-log format of Figure 4 is used to spread out the variations for small values of x where the accelerations are large with resulting rapid changes in velocity. It will be noted from Figure 4 that large increases in nozzle length are required for modest increases in particle velocity when the reduced velocities approach unity. The larger values of reduced velocities with increasing γ that were noted previously in Figure 2 can be seen to result in significantly smaller values of x for the same reduced particle velocity. The optimum streamwise variation of the dimensionless parameters v_s and v_p, together with gas stream Mach number, M_s, and nozzle area ratio, A/A*, are also shown in Figure 5 using linear coordinates for the case of $\gamma = 7/5$.

Some typical variations of nozzle parameters (d_s/d_s^*, v_p, v_s, and M_s) with the dimensional distance, s (in inches), downstream from the particle injection point are presented in Figure 6 through 9 for high-temperature air nozzles that maximize the particle acceleration. The stagnation conditions of the air were taken to be : 68 atmospheres pressure, 5913°R temperature, and 0.458 lbm/ft^3 density. The value of the ratio of specific heats, γ, was taken to be equal to 5/4, which is representative of high-temperature air in an arc. The particles were magnesium oxide with a density, ρ_p, equal to 223.5 lbm/ft^3 and particle diameters, d_p, equal to 5, 50, and 500 microns. The particle drag coefficient, C_D, was constant and equal to 1.0 for most of the curves; a value of $C_D = 2.0$ was also used with the 50 micron case to indicate the effect of changing particle drag coefficient. The results of Figures 6 through 9 show that there exists an optimum nozzle for each particle size and that as particle size increases the length of the optimum nozzle to achieve a given particle velocity also increases. This result is a consequence of the fact that the ratio of inertial to drag force is proportional to λ which, in turn, is proportional to particle diameter. Thus,

it is more difficult to accelerate large particles by a gas stream than it is to accelerate the smaller ones. In general, changing those parameters which decrease λ will result in shorter optimum nozzles for a given particle velocity or for obtaining higher particle velocities from a given nozzle length. From the definition of λ it can be shown that decreasing particle size and particle density as well as increasing particle drag coefficient and stagnation density of the gas will decrease λ.

A meaningful comparison can be made with the results of the present investigation and some recently published calculations of Reference 2 for the case of a conical nozzle. This comparison is shown in Figure 10 where the particle velocity, V_{p_f}, at the exit of the nozzle is plotted as a function of particle diameter d_p. The stagnation conditions of the flow were: 75 atmospheres pressure, 2000 Btu/lbm enthalphy. The maximum stream velocity, V_m, for this value of stagnation enthalpy was found to be 10,000 feet per second when using the energy equation for air given in Reference 5 as

$$V_s = 223.6\ (h_t - h)^{1/2} \tag{22}$$

where the velocity is given in feet per second and the enthalpies are in Btu/lbm. Setting h equal to zero in equation (22) gives

$$V_m = 223.6\ (h_t)^{1/2} \tag{23}$$

The nozzle length was 17.25 feet and both sets of calculations assumed a constant drag coefficient $C_D = 1.0$. The differences in ratio of specific heats for the two sets of calculations ($\gamma = 1.22$ in Reference [2] and $\gamma = 1.25$ for the optimum nozzle) are deemed insignificant. It will be noted from Figure 10 that whereas for small particle sizes only minor improvements in particle velocity are possible (approximately 800 feet per second increase over 6000 feet per second for 50 micron diameter particles), significant increases are possible

for the larger particle sizes (4040 feet per second compared to approximately 2400 feet per second for 1500 micron diameter particles). It should be noted that the stagnation conditions chosen for this calculation do not represent the current state-of-the-art in plasma arc facilities.

Another method for increasing the particle velocities is to increase the value of the maximum stream velocity, V_m, of the gas. From equation (23) it is clear that the stagnation enthalpy of the gas must be increased. This effect is illustrated by the upper curve of Figure 10 which shows the substantial increases of particle velocities in an air arc facility with a stagnation enthalpy of 6000 Btu/lbm (at 190 atmospheres stagnation pressure in order to keep λ essentially invariant between the two sets of calculations). For this case the calculations give (for $C_D = 1.0$) particle velocities of 11,800 and 7,000 feet per second for particle diameters of 50 and 1,500 microns, respectively. Enthalpies at this pressure level appear to be an attainable goal for the near future (Reference 6). Thus, a plasma arc with an appropriately optimized nozzle appears to offer the possibility of significantly improved simulation of the high heating rates simultaneously with the high particle velocities and longer testing times required for advanced vehicles.

Section 4

CONCLUSIONS

The problem of determining the streamwise variation of gas stream velocity in a nozzle such that the acceleration of particles within the gas stream is everywhere a maximum has been reduced to a quadrature. The assumptions for which this solution has been obtained are that the following parameters of the problem are constants: ratio of specific heats and stagnation conditions of the gas; mass, size and drag coefficient of the particles. The quadratures (in non-dimensional form) have been evaluated for three values of the ratio of specific heats, namely, 5/4, 7/5, and 5/3.

The results of this investigation indicate that a plasma arc with an appropriately optimized nozzle appears to offer the possibility of significantly improved simulation of the high heating rates simultaneously with high particle velocities and longer testing times required for advanced hypersonic vehicles. Furthermore, the particle velocities for a fixed length of such an optimized nozzle can be increased by (a) increasing the gas stagnation enthalpy, the gas stagnation density, and the particle drag coefficient; and (b) decreasing the particle size and the particle density.

Comparative calculations, assuming a particle drag coefficient $C_D = 1.0$, for 17.25 foot long nozzles operating at 2000 Btu/lbm stagnation enthalpy and 75 atmospheres stagnation pressure show that whereas a conical nozzle may achieve particle velocities of 6,000 and 2,400 feet per second for particle diameters of 50 and 1,500 microns, respectively, an optimized nozzle may achieve 6,800 and 4,040 feet per second for the same particle sizes of 50 and 1,500 microns diameter, respectively. Increasing the stagnation enthalpy to 6,000 Btu/lbm and the

stagnation pressure to 190 atmospheres, the optimized nozzle may achieve particle velocities of 11,800 and 7,000 feet per second for particle diameters of 50 and 1,500 microns, respectively.

Recommendations for future work include first, the extension of the present analysis to include arbitrary values of the ratio of specific heats by utilizing numerical techniques for the evaluation of the quadrature relating stream velocity to the distance downstream from the particle injection point. Parametric calculations could then be effeciently performanced to determine quantitatively the effects of particle size, stagnation conditions, length of device, working fluid (gamma and molecular weight) on achievable particle velocities. Next, investigate numerical techniques to solve the nozzle optimization problem with fewer assumptions. In particular, the assumption of constant drag coefficient would be removed and a variation of drag coefficient with, at least, Reynolds number and (if sufficient data is available) with Mach number would be used in the determination of particle forces. Another approximation that can be removed is that the gas is calorifically perfect with constant γ and constant composition to include the effects of chemical recombination and the variation of specific heats with temperature. Finally, the potential capabilities of existing plasma arc facilities as particle accelerators should be compared. The characteristics of existing plasma arcs should be gathered and the results of the present optimization study, or the recommended extensions, should be applied to determine the ability of these arcs to accelerate particles when modified by the addition of optimum nozzles. The merit of this recommended work would be either to make it possible to utilize existing facilities to achieve velocities required by advanced vehicles by modification or extension of the hardware, or to lay the foundation for the design of new hardware, such as heaters with higher enthalpy-pressure capability.

Section 5

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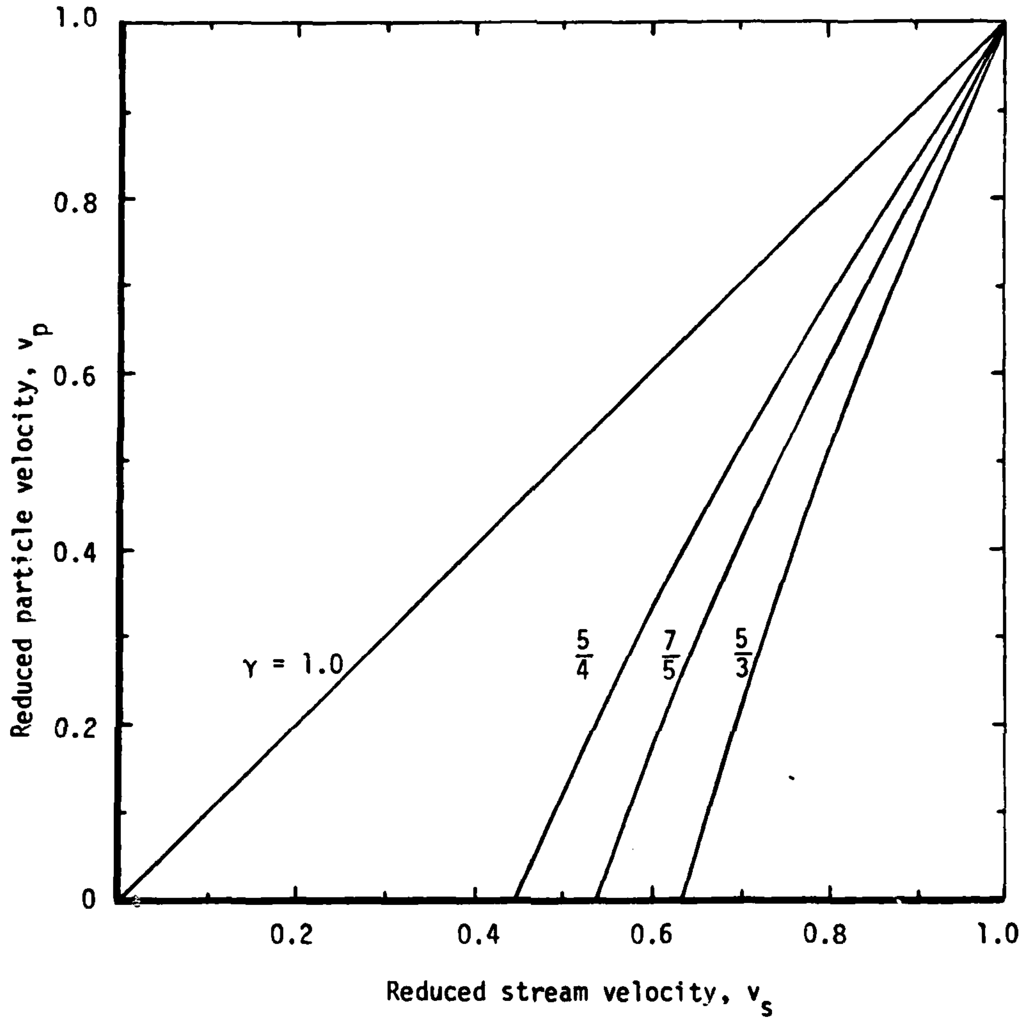

Figure 1. RELATION BETWEEN REDUCED VELOCITIES FOR MAXIMUM PARTICLE ACCELERATION

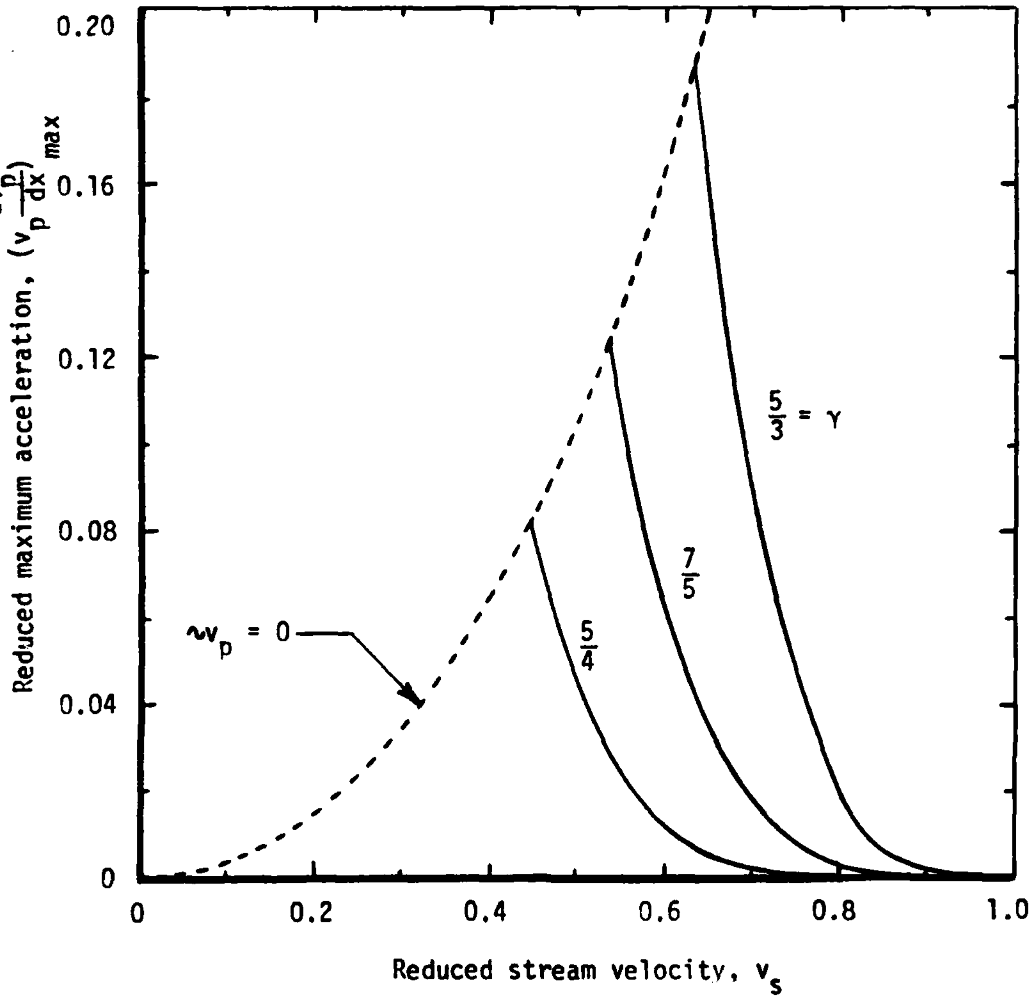

Figure 2. VARIATION OF REDUCED MAXIMUM PARTICLE ACCELERATION WITH REDUCED STREAM VELOCITY

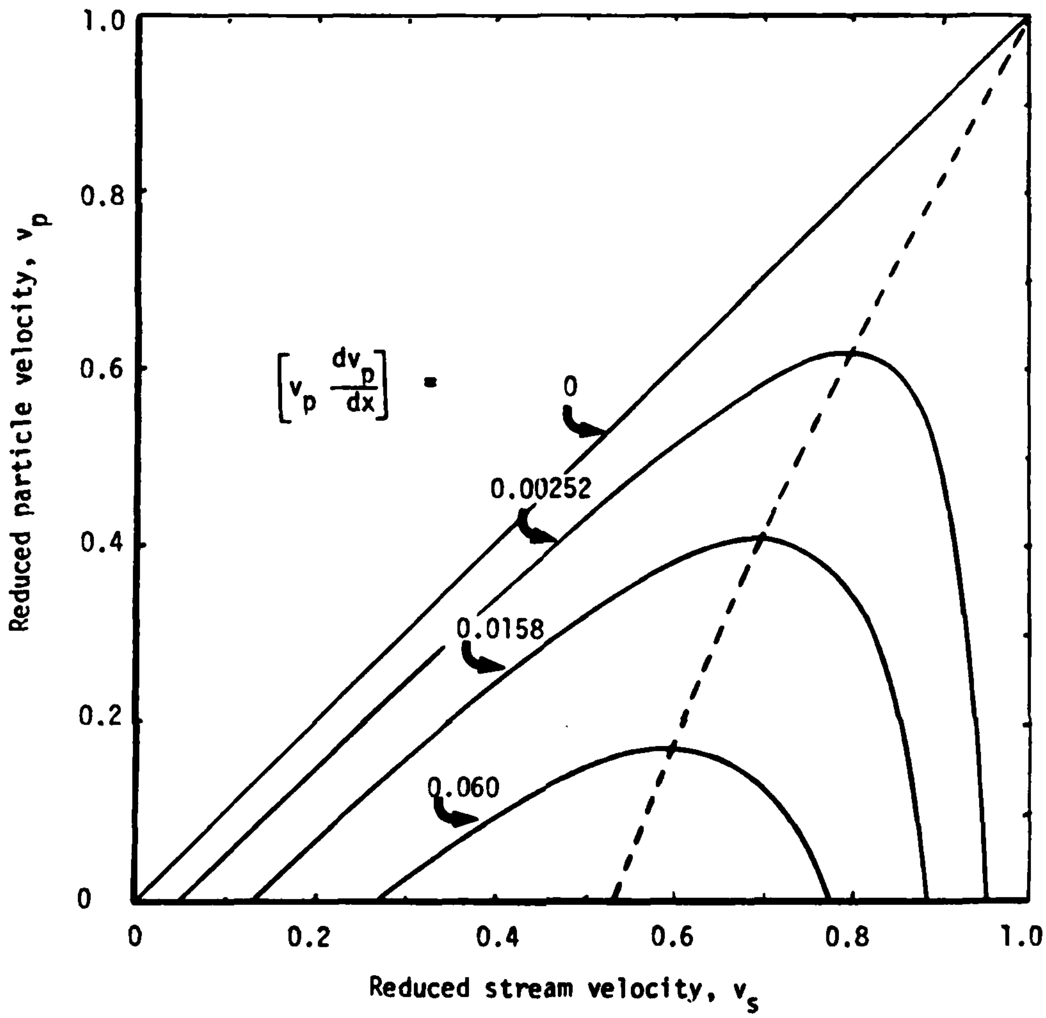

Figure 3. VARIATION OF REDUCED PARTICLE VELOCITY WITH REDUCED STREAM VELOCITY FOR DIFFERENT VALUES OF NON-DIMENSIONAL PARTICLE ACCELERATION; $\gamma = 7/5$

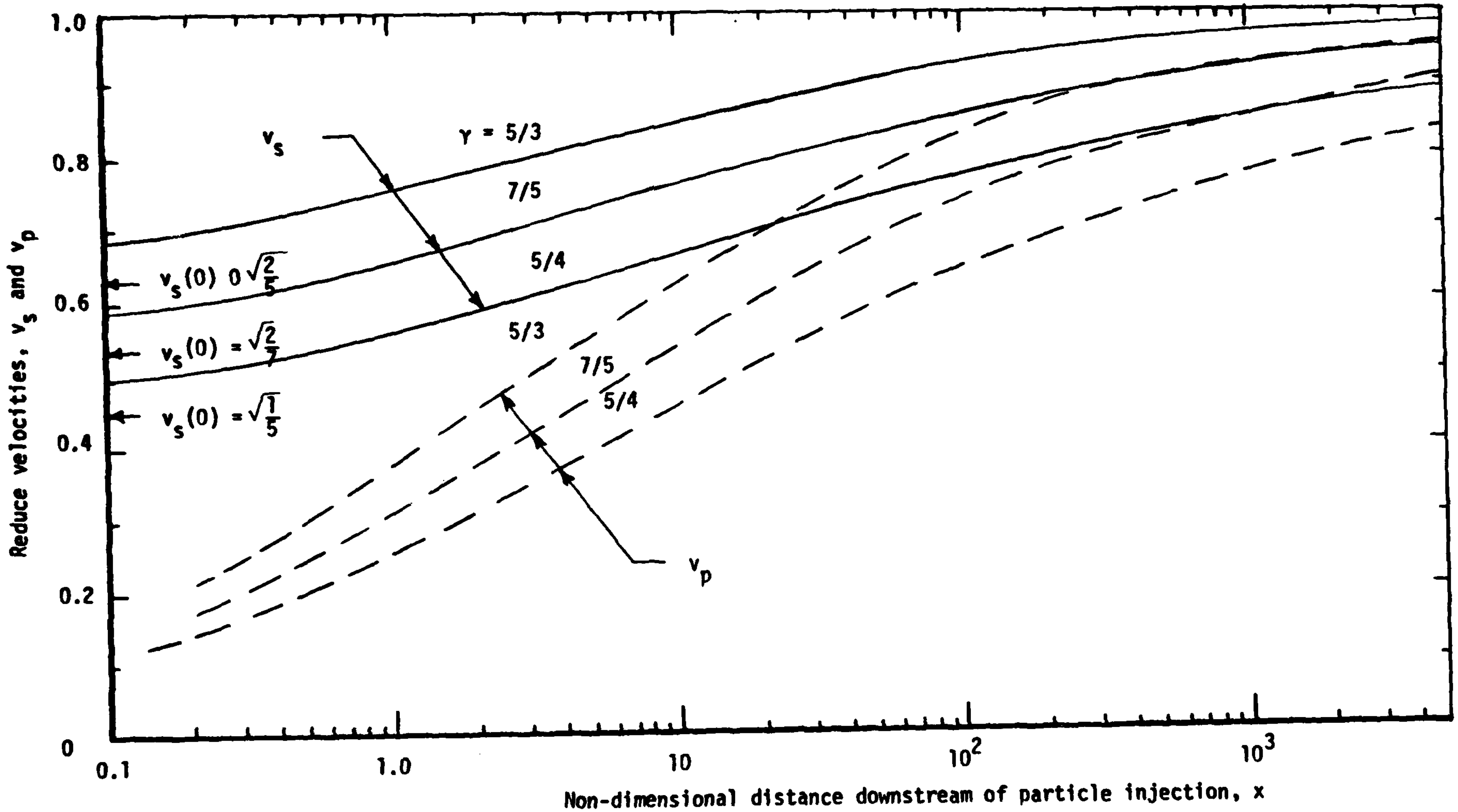

Figure 4. VARIATION OF REDUCED VELOCITIES WITH NON-DIMENSIONAL DISTANCE DOWNSTREAM FROM POINT OF PARTICLE INJECTION FOR MAXIMUM PARTICLE ACCELERATION

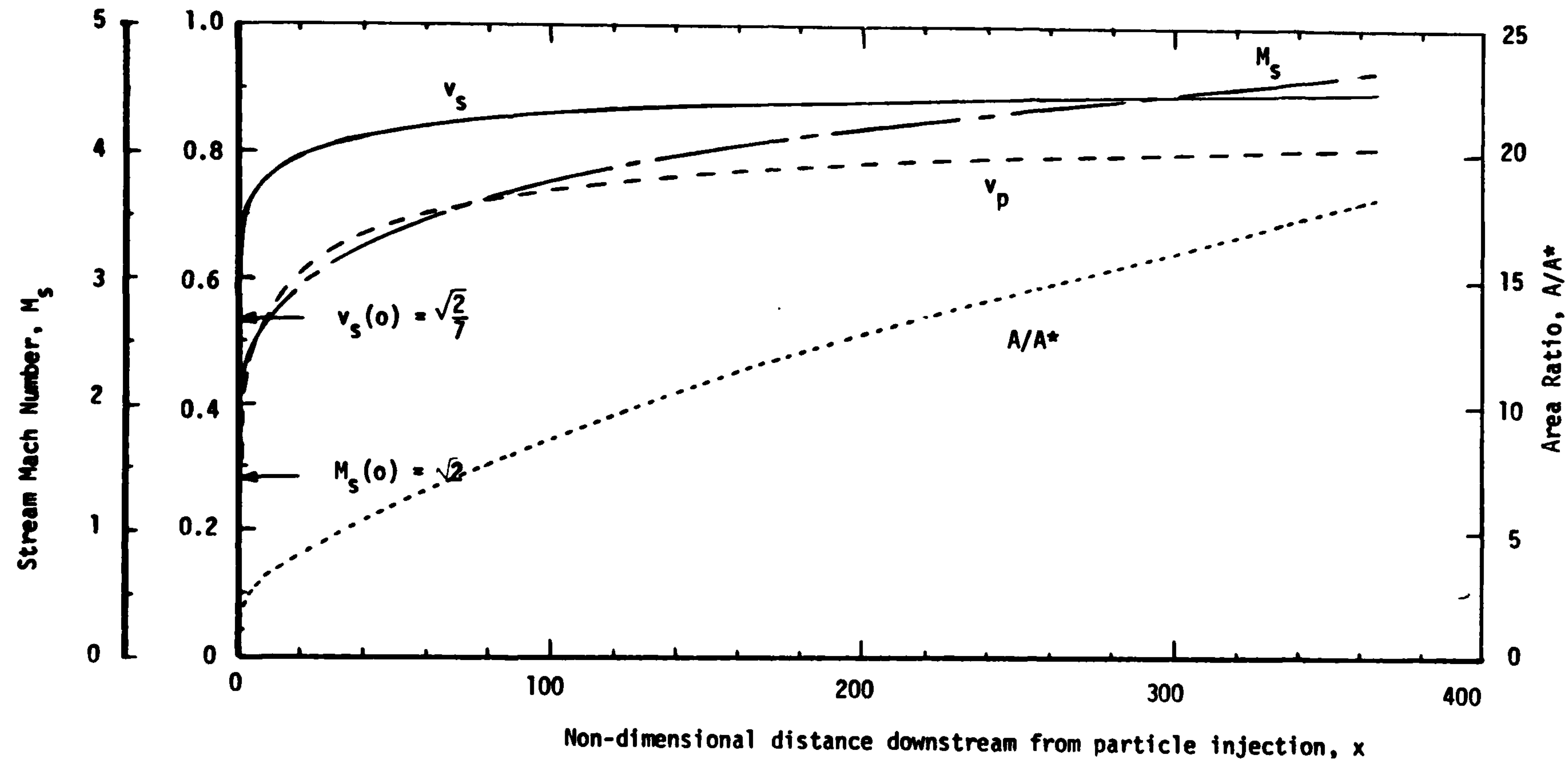

Figure 5. OPTIMUM STREAMWISE VARIATION OF NOZZLE PARAMETERS; $\gamma = 7/5$

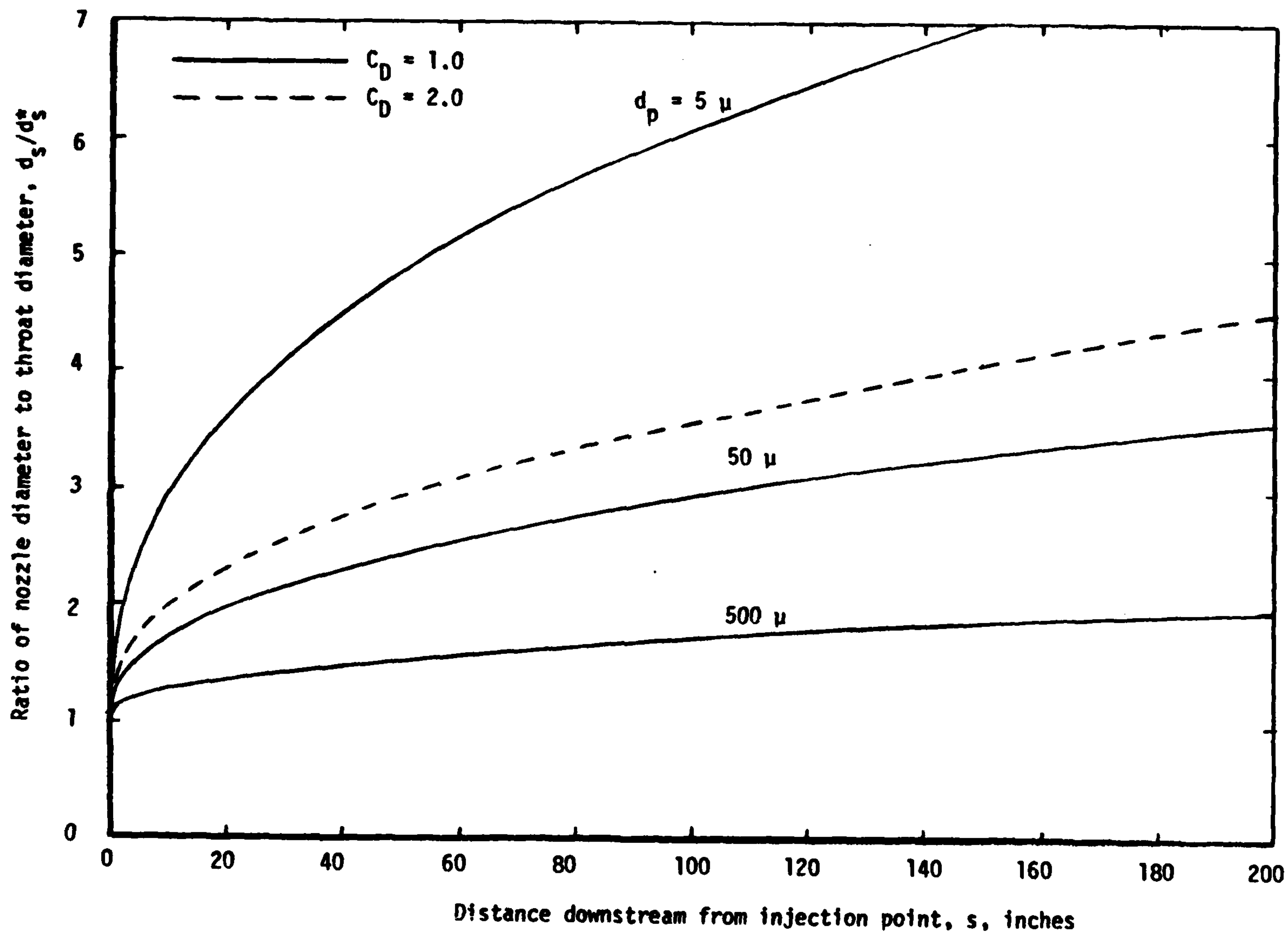

Figure 6. OPTIMUM NOZZLE DIAMETER RATIO FOR DIFFERENT PARTICLE SIZES; $\gamma = 5/4$, $p_t = 68$ ATMOSPHERES, $T_t = 5913°R$

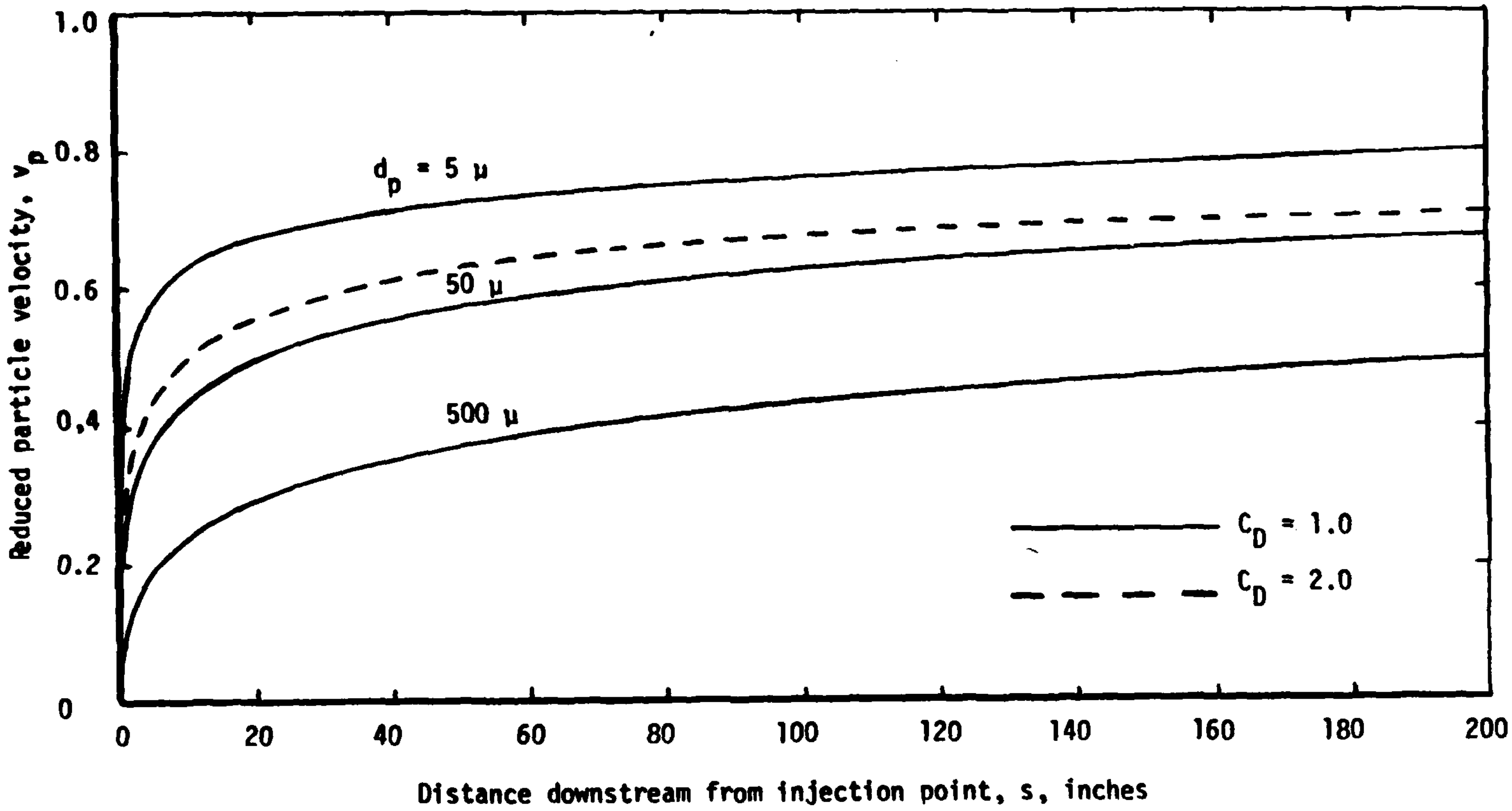

Figure 7. OPTIMUM REDUCED PARTICLE VELOCITY FOR DIFFERENT PARTICLE SIZES, $\gamma = 5/4$, p_t = 68 ATMOSPHERES, T_t = 5913°R

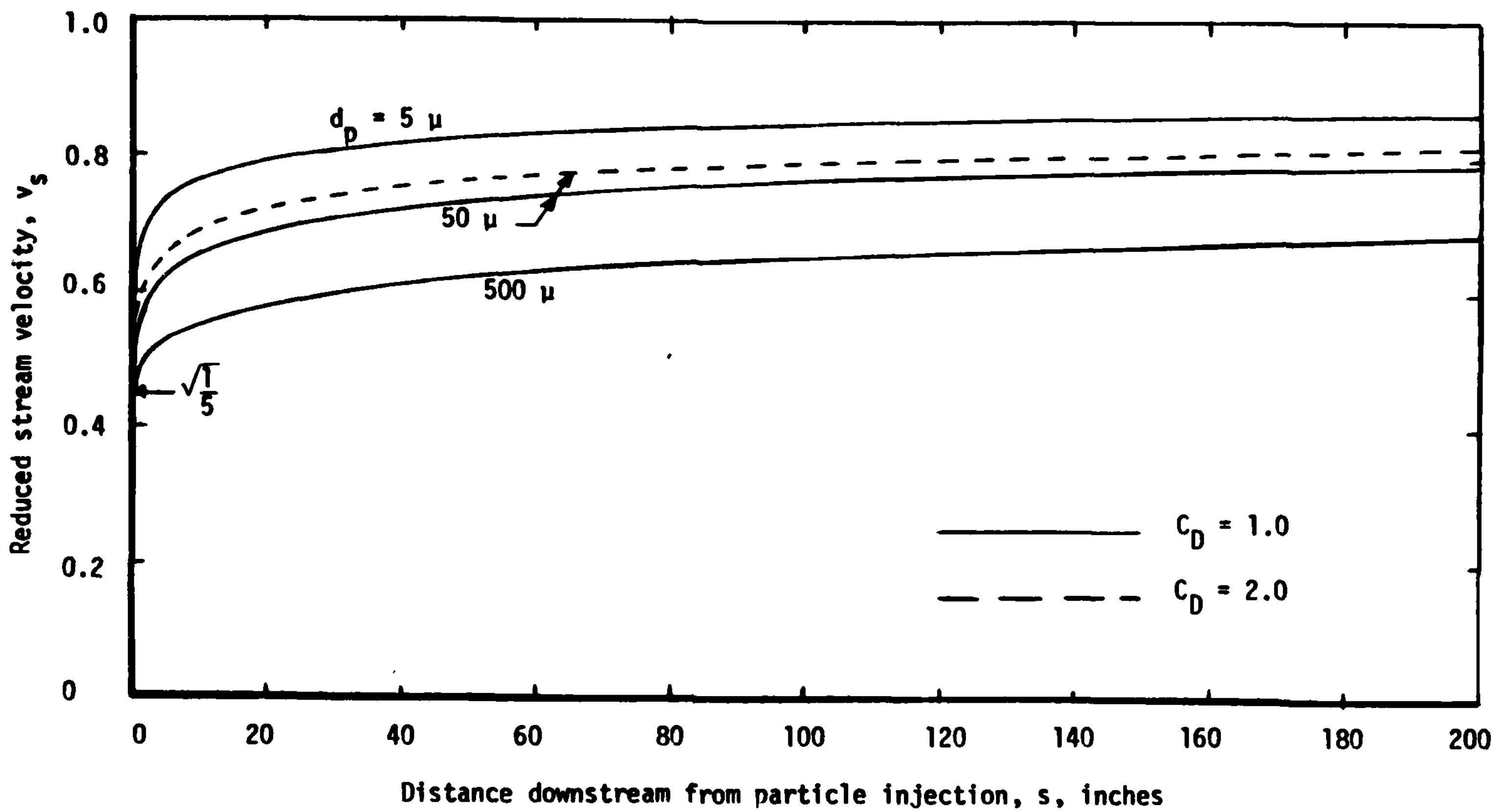

Figure 8. OPTIMUM REDUCED STREAM VELOCITY FOR DIFFERENT PARTICLE SIZES; $\gamma = 5/4$, p_t = 68 ATMOSPHERES, T_t = 5913°R

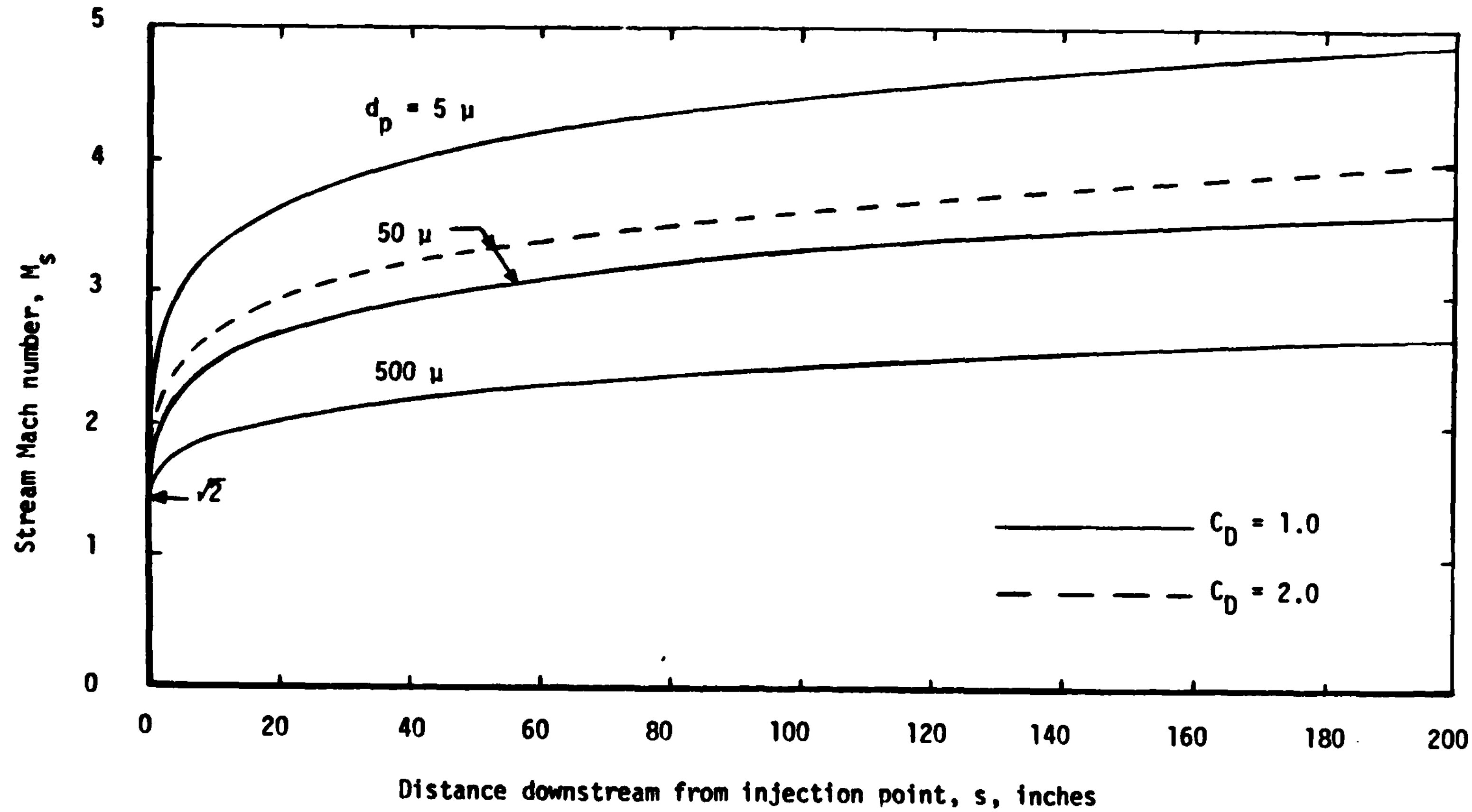

Figure 9. OPTIMUM STREAM MACH NUMBER FOR DIFFERENT PARTICLE SIZES; $\gamma = 5/4$, p_t = 68 ATMOSPHERES, T_t = 5913°R

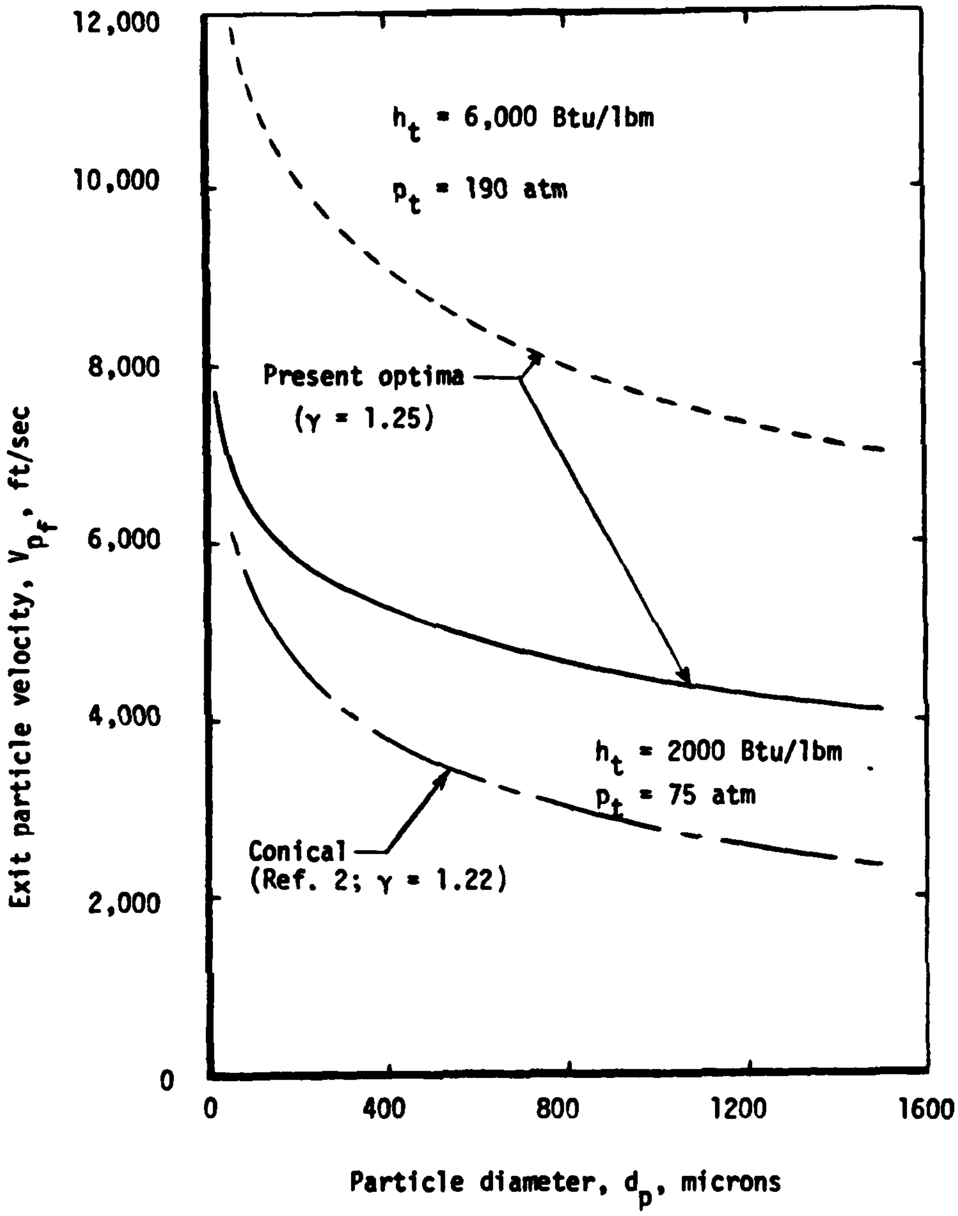

Figure 10. VARIATION OF PARTICLE VELOCITY AT NOZZLE EXIT WITH PARTICLE SIZE FOR A 17.25 FOOT LONG NOZZLE; C_D = 1.0